1. Details of Module and its structure

Subject Name	Physics
Course Name	Physics 03 (Physics Part-1, Class XII)
Module Name/Title	Unit-01, Module-11: Numerical problems based on Capacitors Chapter-02: Electrostatic Potential and Capacitance
Module Id	Leph_10205_eContent
Pre-requisites	Electrostatic force, Coulomb's laws of electrostatics, Electrostatic potential and potential difference, Superposition principle, Grouping of capacitors, Energy stored in a capacitor, capacitance of a parallel plate capacitor, Effect of di-electric on the various parameters of a capacitor.
Objectives	After going through this lesson, the learners will be able to: Apply their knowledge to check the quantitative effect of the dimensions of the capacitor, the quantitative effect of filling the gap between the plates of a capacitor by a di-electric on the charge stored, capacitance, potential difference and electric field between the plates. Apply their understanding of capacitors in combination to determine to the potential , energy and capacitance
Keywords	Numerical on Capacitance, Capacitors, polarization of the di-electric, energy stored in a capacitor

2. Development Team

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1. UNIT SYLLABUS

Chapter 1

Electric charges, conservation of charges, Coulomb’s law- force between two point charges, force between multiple charges, superposition principle and continuous charge distribution
Electric field, Electric field due to a point charge, Electric field lines, Electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.
Electric flux, statement of Gauss’s theorem and it’s applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell

Chapter 2

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges, equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole an electrostatic field.
Conductors and insulators, free charges and bound charges inside a conductor, di-electrics and electric polarisation, Capacitors and capacitance, combination of capacitors in series and parallel, capacitance of a parallel plate capacitor with and without di-electric medium between the plates, energy stored in a capacitor.

2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS

The above unit is divided into 11 modules for better understanding. **11 Modules**

Module 1	Electric charge Properties of charge Coulombs' law Characteristics of coulomb force Constant and the intervening medium numerical
Module 2	Forces between multiple charges Superposition Continuous distribution of charges numerical
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Module 10	Capacitance of a parallel plate capacitor with and without dielectric medium between the plates Energy stored in a capacitor
Module 11	Typical problems on capacitors

MODULE 11

3. WORDS YOU MUST KNOW

Let us recollect the words we have been using in our study of this physics course.

- Electric Charge: Electric charge is an intrinsic characteristic, of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions.

- Conductors: Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called *conductors*. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are all conductors of electricity.

- Insulators: Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called *insulators*.

- Point Charge: When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as *point charges*.

- Coulomb's Force: It is the electrostatic force of interaction between the two point charges.

- Linear charge density: The linear charge density λ is defined as the charge per unit length.
- Surface charge density: The surface charge density σ is defined as the charge per unit surface area.

- Volume charge density: The volume charge density ρ is defined as the charge per unit volume.

- *Superposition Principle*: For an assembly of charges q_1, q_2, q_3, ..., the force on any charge, say q_1, is the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by the Coulomb's law for two point charges.

- Electric Field: A region around a charge particle or object within which a force would be experienced by charge particle or object.

- Source and test charge: The charge, which is producing the electric field, is called a *source charge* and the charge, which tests the effect of as source charge, is called a *test charge.*

- Uniform Field: A *uniform electric field* is one whose magnitude and direction is same at all points in space and it will exert same force of a charge regardless of the position of charge.

- Non uniform field: we know that electric field of point charge depends upon location of the charge. Hence has different magnitude and direction at different points. We refer to this field as non-uniform electric field

- Principle of superposition of fields: Electric field intensity E at any point P due to all n point charges will be equal to the vector sum of electric field intensities E_1 ,E_2, E_3……E_n produced by individual charges at the point P. Hence $E = E_1 + E_2 + \ldots + E_n$

- Torque: Torque is the tendency of a force to rotate an object about an axis.

- Electric field lines: An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point.

- Surface charge density in terms of area element: The surface charge density σat the area element Δs is given by $\sigma = \frac{\Delta Q}{\Delta s}$

- Area vector: The area element vector ΔS at a point on a closed surface equals $\Delta S\,\hat{n}$ where ΔS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.

- Gaussian surface: The closed surface that we need to choose for applying Gauss's law to a particular charge distribution is called the Gaussian surface.

- Gauss's law: The flux of the electric field through any closed surface *S* is $1/\varepsilon_0$ times the total charge enclosed by that surface.

- Electrostatic force: Like charges repel each other and unlike charges attract each other, this force of attraction, or repulsion, between two charges, is called electrostatic force.

- Coulomb's Law of electrostatics: The force of attraction, or repulsion, between two point charges, is directly proportional to the product of the magnitude of the two charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.

- Electrostatic potential: It is the amount of work done in moving a unit positive charge, from infinity to that point, in the electric field, without accelerating it.

- Principle of superposition: If we have a collection of point charges, the force on any one of them, is the vector sum of the electrostatic forces exerted by all the other point charges.

- Capacitance: It measures the ability to store charge and it is equal to the amount of charge required to have a potential difference of 1V between the plates of the capacitor.

- Common Potential: When two capacitors charged to different potential are connected the charge will flow from higher potential to the one at lower potential till the both acquire the same potential, this potential acquired is called the common potential.

- Series grouping of capacitors:

  In series combination,

i) The charge on each capacitor is same.

ii) The reciprocal of equivalent capacitance, of a series combination, is equal to the sum of, the reciprocals, of the individual capacitances.

- Parallel grouping of capacitors:

  In parallel grouping

i) The equivalent capacitance is equal to the sum of the individual capacitances.

ii) The potential difference across each capacitor is same.

iii) The charge on each capacitor is proportional to its capacitance.

- Common potential: When conductors charged to different potential are connected then charge will flow from a body at higher potential to the one at lower potential till both reach at the same potential called common potential.

4. INTRODUCTION

We have considered in the earlier modules that a capacitor is a system of two conductors separated by an insulator. The conductors have charges; say Q_1 and Q_2, and potentials V_1 and V_2. Usually, in practice, the two conductors have charges Q and – Q, with potential difference $V = V_1 – V_2$ between them. We learnt that we can change the net value of capacitance by combining them in different ways. This is required in practical circuits for use of capacitors in daily life

It is to fulfil this requirement that we have learnt about capacitors and their different ways of grouping, let us try to solve some typical problems on capacitors

5. FORMULAE USED

Q is Charge

V is Potential

C is Capacitance

- **Q = CV**

- **Equivalent capacitance of a series combination is given by**

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Equivalent capacitance of a parallel combination is given by**

$$C_p = C_1 + C_2 + \cdots C_n$$

- **Common potential V is given by**

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

- **Capacitance of a parallel plate capacitor**

$$C = Q / V = \frac{A\epsilon_0}{d}$$

- **Capacitance of a capacitor filled with a di- electric C is given by**

$$\frac{C}{C_0} = K$$

- **Energy stored in a capacitor**

$$\mathcal{U} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- **di-electric constant of the slab is K (entire space between the plates is filled with the di-electric**)

Battery disconnected from the capacitor	Battery kept connected across the capacitor
$Q = Q_0$ (constant)	$Q = KQ_0$
$V = \frac{V_0}{K}$	$V = V_0$ (constant)
$E = \frac{E_0}{K}$	$E = E_0$ (constant)
$C = K\,C_0$	$C = K\,C_0$
$U = \frac{U_0}{K}$	$U = K\,U_0$

6. TYPICAL NUMERICAL PROBLEMS

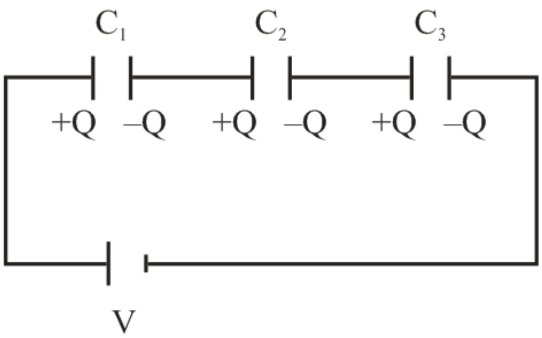
EXAMPLE

If three capacitors each of 9 pf are connected in series.

(i) What is the total capacitance of the combination?

(ii) What is the change on each capacitor?

(iii) What is the potential difference across each capacitor of the combination is connected to a power supply of 120 V?



SOLUTION:

(i) As capacitors are connected in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$$C = 3pF$$

(ii) As in series combination charge on each capacitor is equal and it is equal to the charge drawn from the battery.

$$Q_1 = Q_2 = Q_3 = Q = CV = 3 \times 10^{-12} \times 120$$

$$= 360 \times 10^{-12} C$$

(iii) $Q_1 = C_1 V_1$ So $V_1 = \frac{Q_1}{C_1}$

$$V_1 = \frac{360 \times 10^{-12}}{9 \times 10^{-12}} = 40V$$

$$V_2 = \frac{Q_2}{C_2} = \frac{360 \times 10^{-12}}{9 \times 10^{12}} = 40V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{360 \times 10^{-12}}{9 \times 10^{12}} = 40V$$

EXAMPLE

The equivalent capacitance of the combination between A and B in figure given below is $4\mu F$.

(a) Calculate capacitance of the capacitor C.

(b) Calculate the charge on each capacitor if a 10V battery is connected across the terminals A and B.

(c) What will be the potential drop across each capacitor?

SOLUTION:

(a) $C_1 = 20\mu F,\ C_2 = C$

C_1 and C_2 are connected in series

$$\frac{1}{4} = \frac{1}{20} + \frac{1}{C}$$

$$\frac{1}{C} = \frac{1}{4} - \frac{1}{20} = \frac{5-1}{20} = \frac{4}{20}$$

So $C = \frac{20}{4} = 5\mu F$

(b) Charge on each capacitor

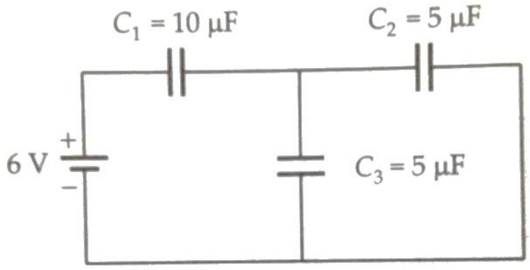
$$q = C_{AB}V = 4 \times 10^{-6} \times 10 = 40 \times 10^{-6} C$$

(c) P.D. across $20\mu F = \frac{q}{C} = \frac{40\times10^{-6}}{20\times10^{-6}} = 2V$

P.D. across $5\mu F = \frac{q}{C} = \frac{40\times10^{-6}}{5\times10^{-6}} = 8V$

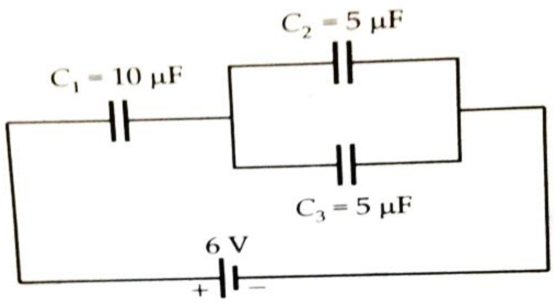
EXAMPLE

Three capacitors C_1, C_2 and C_3 are connected to a $6V$ battery, as shown in Figure. Find the charges on the three capacitors.



SOLUTION:

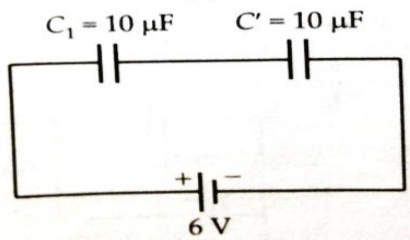
The given arrangement is equivalent to the arrangement shown in figure.



(*a*)

Clearly, C_2 and C_3 are in parallel. Their equivalent capacitance is

$$C = C_2 + C_3 = 5 + 5 = 10\ \mu F$$



(*b*)

Now C_2 and C' form a series combination, as shown in figure. Their equivalent capacitance is

$$C = \frac{C_1 C}{C_1 + C} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Charge drawn from the battery,

$$q = CV = 5\ \mu F \times 6V = 30\ \mu C$$

Charge on the capacitor $C_1 = q = 30\ \mu C$.

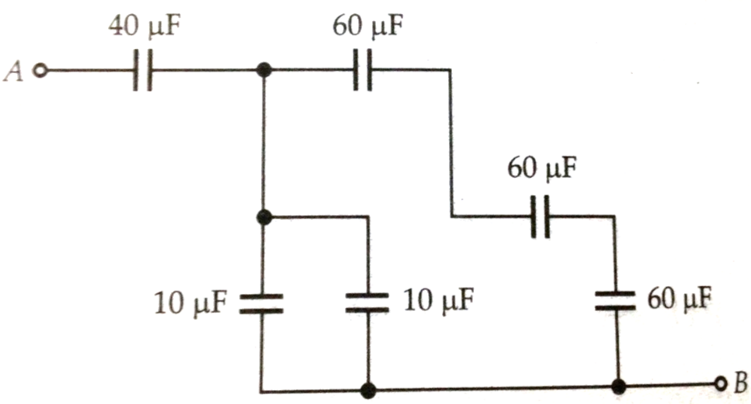
Charge on the parallel combination of C_2 and $C_3 = 1 = 30\ \mu C$.

As C_2 and $C_3$are equal, so the charge is shared equally by the two capacitors.

Charge on $C_2 = charge\ on\ C_3 = \frac{30}{2} = 15\ \mu C$.

EXAMPLE

Find the equivalent capacitance of the combination of capacitors between the



points A and B as shown in Figure. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and B.

SOLUTION:

Here three capacitors of $\mathbf{60\mu F}$ each are connected in series. Their equivalent capacitance $\boldsymbol{C_1}$ is given by

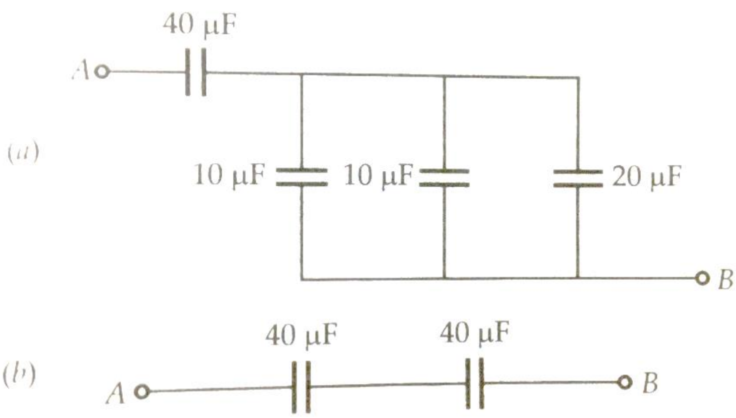
$$\frac{1}{C_1} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$$

or $$C = 20\ \mu F$$

The given arrangement now reduces to the equivalent circuit shown in Figure.

Clearly, the three capacitors of $\mathbf{10\mu F, 10\mu F}$ and $\mathbf{20\mu F}$ are in parallel. Their equivalent capacitance is

$$C_2 = 10 + 10 + 20 \times 40\ \mu F$$



Now the circuit r educes to the equivalent circuit shown in Figure. We have two capacitors of $\mathbf{40\mu F}$ each connected in series. The equivalent capacitance between $\boldsymbol{A}$ and $\boldsymbol{B}$ is

$$C = \frac{40 \times 40}{40 + 40} = 20\mu F$$

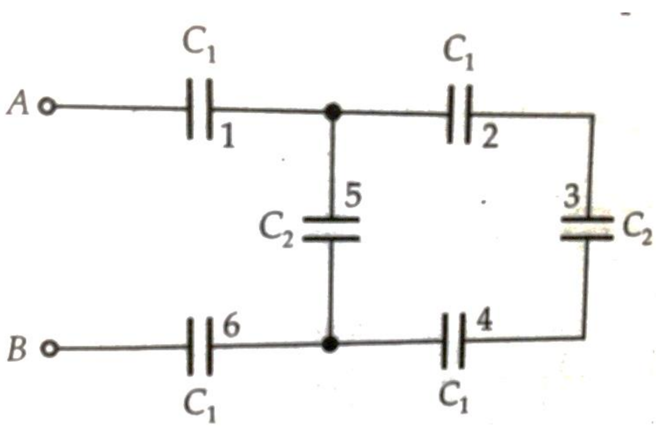
Given $\quad V = 100V$

$\therefore$ Charge, $q = CV = 20\mu F \times 100V$

$$= 2000\mu C = 2mC$$

EXAMPLE

If $C_1 = 3pF$ and $C_2 = 2pF$, calculate the equivalent capacitance of the given network between points A and B.



SOLUTION:

Clearly, capacitors 2, 3 and 4 form a series combination. Their total capacitance $\boldsymbol{C'}$ is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{7}{6}$$

$$\therefore \qquad C' = \frac{6}{7}pF$$

The capacitance C’ forms a parallel combination with capacitor 5, so their equivalent capacitance is

$$C" = C' + C_2 = \frac{6}{7} + 2 = \frac{20}{7}pF$$

The capacitance C” forms a series combination with capacitors 1 and 6. The equivalent capacitance C of the entire network is given by

$$\frac{1}{C} = \frac{1}{C"} + \frac{1}{C_1} + \frac{1}{C_1} = \frac{7}{20} + \frac{1}{3} + \frac{1}{3} = \frac{61}{60}$$

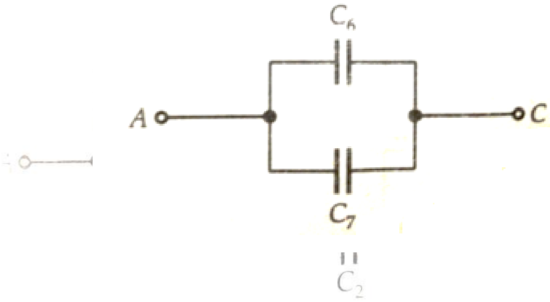
$$\therefore \qquad C = \frac{60}{61}pF$$

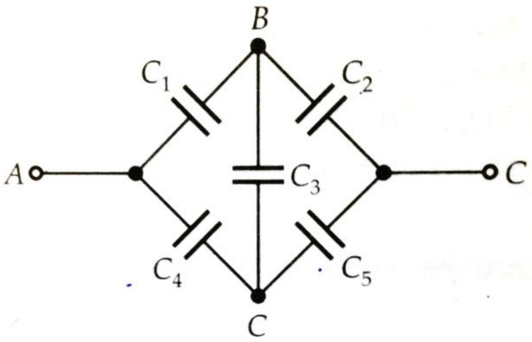
EXAMPLE

Five capacitors of capacitance $10\mu F$ each are connected with each other, as shown in Figure. Calculate the total capacitance between the points A and C.

SOLUTION:

The given circuit can be redrawn in the form of a Wheatstone bridge as shown in Figure.

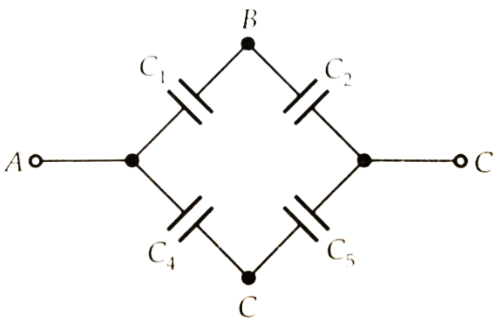




As $\boldsymbol{C_1 = C_2 = C_4 = C_5}$,

Therefore, $\frac{\boldsymbol{C_1}}{\boldsymbol{C_2}} = \frac{\boldsymbol{C_4}}{\boldsymbol{C_5}}$

Thus the given circuit is a balanced Wheatstone bridge.



A Wheatstone bridge is an electrical circuit of three known and one unknown resistance used to measure the value of unknown electrical resistance by balancing.

In a balanced Wheatstone bridge the ratio of any pair of resistances is equal to the ratio of the other two.

The Wheatstone bridge illustrates the concept of a difference measurement, which can be extremely accurate. Variations on the Wheatstone bridge can be used to measure capacitance.

So the difference across the ends of capacitor $\boldsymbol{C_3}$ is zero. Capacitance $\boldsymbol{C_3}$ is ineffective. The given circuit reduces to the equivalent circuit shown in Figure.

Capacitors C_1 and C_3 form a series combination of equivalent capacitance C_6 given by

$$C_6 = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Similarly, C_4 and C_5 form a series combination of equivalent capacitance C_7 given by

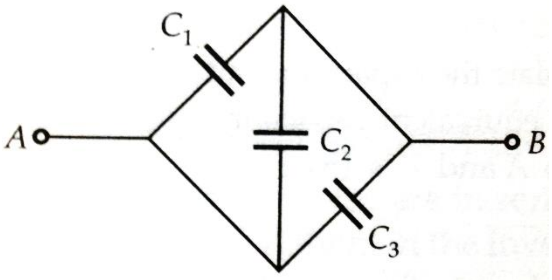
$$C_7 = \frac{C_4 \times C_5}{C_4 + C_5} = \frac{10 \times 10}{10 + 10} = 5\mu F$$

As shown in Figure, C_6 and C_7 form a parallel combination. Hence the equivalent capacitance of the network is given by:

$$C = C_6 + C_7 = 5 + 5 = 10\ \mu F$$

TRY YOURSELF EXERCISE

In Figure, $C_1 = 1\mu F, C_2 = 2\mu F$ and $C_3 = 3\mu F$. Find the equivalent capacitance between points A and B.



Answer 6μF

EXAMPLE

A network of four 10 μF capacitors is connected to a 500 V supply, as shown in Fig. . Determine

(a) The equivalent capacitance of the network and

(b) The charge on each capacitor.

(Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential).

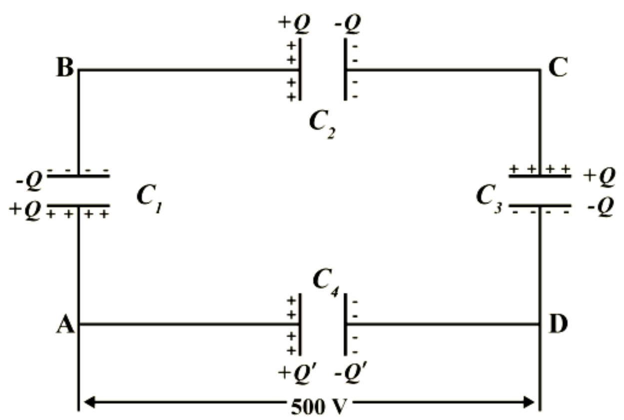
SOLUTION

(a) In the given network, C_1, C_2 and C_3 are connected in series. The effective capacitance C' of these three capacitors is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For $C_1 = C_2 = C_3 = 10\ \mu F$, C' = (10/3) μF. The network has C' and C4 connected in parallel. Thus, the equivalent capacitance C of the network is

$$C = C' + C_4 = \left(\frac{10}{3} = 10\right)\mu F = 13.3\mu F$$



(b) Clearly, from the figure, the charge on each of the capacitors, C_1, C_2 and C_3 is the same, say Q. Let the charge on C_4 be Q'. Let the charge on C_4 be Q'. Now, since the potential difference across AB is Q/C_1, across BC is Q/C_2, across CD is Q/C_3, we have

$$\frac{Q}{C} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500V$$

Also, $Q'/C_4 = 500$ V.

This gives for the given value of the capacitances,

Q = 500 V × $\frac{10}{3}\mu F = 1.7 \times 10^{-3} C\ and$

Q' = 500 V × 10μF = 5.0×10^{-3} C

EXAMPLE

A 900 pF capacitor is charged by 100V battery.

a) What is the electrostatic energy stored by the capacitor ?

The capacitor is disconnected from the battery and connected to another 900pF capacitor.

b) What is the electrostatic energy stored in the system?

SOLUTION:

a) The charge on the capacitor is

$Q1= C_1V_1 = 900 \times 10^{-12} \times 100 = 9\times10^{-8}$ C

The energy stored by the capacitor is

$= (1/2)\ C_1V_1^2 = (1/2)\ Q_1V_1 = (1/2)\ 9\times10^{-8} \times 100 = 4.5\times 10^{-6}$ J

Charge stored in the second capacitor is = 0

When two capacitors are connected together, charge will flow from the charged capacitor to the uncharged capacitor till both the capacitors reach common potential V

$V= (C_1V_1 + C_2V_2)/(C_1 + C_2) = (900\times10^{-12} \times 100 + 0)/\ 1800 \times10^{-12} = 50V$

Equivalent capacitance $C = C_1 + C_2 = 900 + 900 = 1800$ pF

The energy stored in the system = $(1/2)C\ V^2 = (1/2)\ 1800 \times10^{-12} \times (50)^2 = 2.25\times10^{-6}$ J

Loss in energy = final energy – initial energy

$= 4.5\times 10^{-6} - 2.25\times10^{-6} = 2.25\times10^{-6}$J

The final energy is only half the initial energy, where has the remaining energy gone?

There is a transient period before the system settles to common potential. During this period a transient current flows from the charged capacitor to the uncharged capacitor. Energy is lost during this time in the form of heat and electromagnetic radiation.

7. SUMMARY

You have learnt:

- To calculate the equivalent capacitance in a grouping: consider that a battery is connected between the two points between which you want to find the equivalent capacitance
- If charge coming from the battery gets divided then capacitors are connected in series or if positive plates of the capacitors are connected at one point and negative plates at the other point then they are connected in parallel.

- Potential difference across the capacitors connected in parallel is same
- If there is no branching, i.e. the charge is not getting divided and the positive of one capacitor is connected to the negative of the next capacitor and so on then they are connected in series.
- Charge on each capacitor connected in series is same.